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Multi Item Fuzzy Inventory Model for Imperfect Items with Uncertain Lead Time and Unreliable Holding Cost; A Geometric Programming Approach

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Abstract: In this paper, a multi item objective function fuzzy inventory model has been derived from imperfect items with uncertain lead time and decreasing holding cost using the geometric programming method. Here lead time is allowed. Lead time and Production cost are taken as a fuzzy number.. The decreasing holding cost is considered to be a continuous function of a product quantity. The total cost is minimized under the restriction of the limitation on the total number of orders using the geometric programming method. The aim of the study is to optimize the optimal order level and total annual cost. Finally, a numerical example and sensitivity analysis in fuzzy environment is given to illustrate this model. Graded mean representation method is used to defuzzify the results.
Keywords: Imperfect Items, Lead time, Unreliable Holding Cost, Trapezoidal fuzzy numbers, Graded Mean Representation method, Geometric Programming.
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1. Introduction

Inventory models with imperfect quality items are studied by researchers in past two decades. Inventories are raw equipment, work-in-process goods and wholly finished goods that are considered to be the part of business resources that are ready or will be ready for a deal. Formulating an appropriate inventory model is one of the major concerns for a trade. Maintaining an inventory (stock of goods) for future sale or use is common in the industry. In order to meet demand on time, companies must keep on hand a stock of goods that is waiting for sale. The purpose of inventory theory is to determine the rules that management can use to minimize the costs associated with maintaining inventory and meeting customer demand. There is a necessity for innovative and successful methods for modeling systems related to inventory management, in the face of uncertainty. Uncertainty exists regarding the control object, as the process of obtaining the necessary information about the object is not always possible. The solution of such complex tasks requires the use of systems analysis, development of a systematic approach to the problem of management in general.

In traditional economic order quantity (EOQ) and economic production quantity (EPQ) model, all items are perfect. It is common to all industries that a certain percent of produced/ordered items are non-conforming (imperfect) quality. In reality, the production process not always produces perfect quality items. Imperfect quality items are ineluctable in an inventory system due to imperfect production process, natural disasters, damages, or many other reasons.

A further module of an inventory model is the lead time, which is the amount of time between the placement of an order to replenish inventory (through either purchasing or producing) and the receipt of the goods into inventory.

Geometric Programming makes use of linearity, separability, convexity, and duality.
The common and powerful classical optimization technique used to solve a class of non-linear optimization programming problems, especially found in engineering design and manufacturing known as geometric programming (GP). In 1961, Duffin, Peterson and Zener tried to solve a wide-ranging of engineering design problems developing the basic theories of geometric programming.

Geometric Programming (GP) is a class of nonlinear optimization with many useful theoretical and computational properties. Although GP in standard form is apparently a non-convex optimization problem, it can be readily turned into a convex optimization problem; hence a local optimum is also a global optimum. GP substantially broadens the scope of Linear Programming (LP) applications, and is naturally suited to model several types of important nonlinear systems in science and engineering.

A geometric program (GP) is a type of mathematical optimization problem characterized by objective and constraint functions that have a distinctive form. Recently developed solution methods can solve even all-encompassing GPs extremely resourceful and reliable.

Abou-EL-Ata and Kotb [1] developed a multi-item EOQ inventory model with varying holding cost under two restrictions: A geometric programming approach. Abou-el-ata, Fergany and El-Wakeel [2] opined that probabilistic multi-item inventory model with varying order cost under two restrictions: a geometric programming approach. Duffin, Peterson and Zener [3] considered that geometric programming –theory and application. Das, Roy and Maiti [4] derived that multi-item inventory model with quantity dependent inventory costs and demand-dependent unit cost under imprecise objective and restrictions: a geometric programming approach. Goyal and Cardenas-Barron [5] investigated a note on: an economic production quantity model for items with the imperfect quality practical approach. Gupta, Kanti Swarup and Man Mohan [6] explained that the concepts and problems in operations research. Hariri and Abou-el-ata [7] rectified that flaw in multi-item production lot-size inventory model with a varying order cost under a restriction: a geometric programming approach. Jung and Klein [8] asserted that the optimal inventory policy under decreasing cost functions via Geometric Programming. Kotb and Fergany [9] presented that multi-item EOQ Model with varying holding: a geometric programming approach. Kotb and Fergany [10] established that multi-item EOQ model with both demand-dependent unit cost and varying leading time via geometric programming. Maloney and Klein [11] discussed that constrained multi-item inventory system**:** an implicit approach. Mandal, Roy and Maiti [12] studied an inventory model of deteriorated items with a constraint: a geometric programming approach. Mandal, Roy and Maiti [13] extended the multi-objective fuzzy inventory model with three constraints: a geometric programming approach. Ojha and Biswal [14] formulated the multi-objective geometric programming problem with weighted mean method. Salameh and Jaber [15] observed that economic production quantity model for items with imperfect quality. Shib Sankar Sana [16] focused that a production-inventory model of imperfect quality products in a three-layer supply chain. Wee, Yu and Chen [17] opined that optimal inventory model for items with imperfect quality and shortage backordering. Werners [18] developed that interactive multiple objective programming subject to flexible constraints. Worral and Hall [19] analyzed that the analysis of an inventory control model using posynomial geometric programming. Zadeh [20] developed the Fuzzy sets. Zimmermann [21] constructed that description and optimization of fuzzy systems.

This paper deals an uncomplicated and practical situation and obtains the optimal solution in multi item inventory model for imperfect items with uncertain lead time and unreliable holding cost using geometric programming method through fuzzy.

We focus here on one significant modified method (i.e) geometric programming method. Based on this modified method we derive an optimal total annual cost. Production cost and lead time are taken as a trapezoidal fuzzy number and for defuzzification we use graded mean representation method.

2. Methodology

2.1 Fuzzy Numbers

Any fuzzy subset of the real line R, whose membership function μ_A satisfied the following conditions, is a generalized fuzzy number $\tilde{A}$.

(i) μ_A is a continuous mapping from R to the closed interval [0, 1].

(ii) $\mu_A = 0, -\infty < x \leq a_1,$

(iii) $\mu_A =$ L(x) is strictly increasing on $[a_1, a_2]$

(iv) $\mu_A = w_A, a_2 \leq x \leq a_3$

(v) $\mu_A =$ R(x) is strictly decreasing on $[a_3, a_4]$

(vi) $\mu_A = 0, a_4 \leq x < \infty$

where $0 < w_A \leq 1$ and a_1, a_2, a_3 and a_4 are real numbers. Also, this type of generalized fuzzy number be denoted as $\tilde{A} = (a_1, a_2, a_3, a_4 : w_A)_{LR}$; When $w_A = 1$, it can be simplified as $\tilde{A} = (a_1, a_2, a_3, a_4)_{LR}$

Trapezoidal Fuzzy Number

A trapezoidal fuzzy number $\tilde{A}$ = (a, b, c, d) is represented with membership function $\mu_{\tilde{A}}$ as:

$$\mu_{\tilde{A}}(x) = \begin{cases} L(x) = \frac{x-a}{b-a}, & when\ a \leq x \leq b; \\ 1, & when\ b \leq x \leq c; \\ R(x) = \frac{d-x}{d-c}, & when\ c \leq x \leq d; \\ 0, & otherwise \end{cases}$$

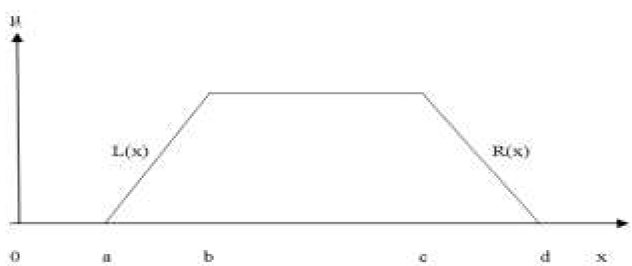

Fig.1: Trapezoidal Fuzzy Number

2.3 The Function Principle

The function principle is used for the operation of addition, subtraction, multiplication and division of fuzzy numbers.

Suppose $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ are two trapezoidal fuzzy numbers, then arithmetical operations are defined as:

1. $\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$

2. $\tilde{A} \otimes \tilde{B}$ = $(a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4)$
3. $\tilde{A} \ominus \tilde{B}$ = $(a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$
4. $\tilde{A} \oslash \tilde{B}$ = $\left(\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1}\right)$
5. $\alpha \otimes \tilde{A}$ = $\begin{cases} (\alpha a1, \alpha a2, \alpha a3, \alpha a4), \alpha \geq 0 \\ (\alpha a4, \alpha a3, \alpha a2, \alpha a1), \alpha < 0 \end{cases}$

2.4 Geometric Programming

2.4.1 Formulation of Multi-Objective Geometric Programming

A multi-objective geometric programming problem can be defined as:

Find $x = (x_1, x_2, ..., x_n)^T$

so as to

$$\min g_{k0}(x) = \sum_{i=1}^{T_{k0}} C_{k0p} \prod_{j=1}^{n} x_j^{b_{k0pj}},\ k = 1,2,...,l$$

subject to the constraints

$$g_i(x) = \sum_{p=1}^{T_i} C_{ip} \prod_{j=1}^{n} x_j^{d_{ipj}} \leq 1,\ i = 1,2,...,m$$

$$x_j > 0,\ j = 1,2,...,n$$

where C_{k0p}, $\forall$ k and p are positive real numbers and d_{ipj} and a_{k0pj} are real numbers for all i, j, k, p.

Here l = number of minimization type objective function

m = number of inequality type constraints and

n = number of strictly positive decision variables.

2.4.2 Weighted Method of Multi-Objective Geometric Programming

The weighted method is the easiest and functional multi-objective optimization which has been extensively applied to attain the optimal solution of multi-objective function within the convex objective space.

According to the number of objective functions, the weights $w_1, w_2, ..., w_l$ are assigned to define a new minimization type objective function Z (w) which can be defined as

$$\min Z(x) = \sum_{k=1}^{l} w_k\, g_{k0}(x)$$

$$= \sum_{k=l}^{l} w_k \left(\sum_{p=1}^{T_{k0}} C_{k0p} \prod_{j=1}^{n} x_j^{b_{k0pj}} \right)$$

$$= \sum_{k=1}^{l} \sum_{i=1}^{T_{k0}} w_k\, C_{k0p} \prod_{j=1}^{n} x_j^{b_{k0pj}}$$

subject to the constraints

$$\sum_{p=1}^{T_i} C_{ip} \prod_{j=1}^{n} x_j^{d_{ipj}} \leq 1,\ i = 1,2,...,m$$

$$x_j > 0,\ j = 1,2,...,n$$

where $\sum_{k=1}^{l_i} w_k = 1,\ w_k > 0,\ k = 1,2,...,l$

2.4.3 Dual Form of Geometric Programming

According to Duffin, Peterson and Zener [3], the given model is transformed to the corresponding dual form of the geometric programming is

$$\max_{w} = \prod_{i=1}^{T_0} \left(\frac{w_k\, C_{k0p}}{w_{0p}} \right)^{w_{0p}} \prod_{i=1}^{m} \prod_{p=1}^{T_i} \left(\frac{w_{i0}\, C_{ip}}{w_{ip}} \right)^{w_{ip}} \prod_{p=1}^{T_i} \lambda\left(w_{ip}\right)^{\lambda\left(w_{ip}\right)}$$

subject to the constraints

$$\sum_{p=1}^{T_0} w_{0p} = 1$$

$$\sum_{i=1}^{m}\sum_{p=1}^{T_i} b_{ipj}\, w_{ip} + \sum_{i=1}^{m}\sum_{p=1}^{T_i} d_{ipj}\, w_{ip} = 0, \quad j = 1,2,\ldots,n$$

$$w_{ip} \geq 0, \ \forall\, p, j$$

where $\sum_{k=1}^{l_i} w_k = 1,\ w_k > 0,\ k = 1,2,\ldots,l$

2.5 Graded Mean Integration Representation Method

If $\tilde{A} = (a_1, a_2, a_3, a_4)$ is a trapezoidal fuzzy number then the graded mean integration representation of $\tilde{A}$ is,

$$p(\tilde{A}) = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6}$$

2.6 Notations and Assumptions

The mathematical model in this paper is derived on the basis of the following notations and assumptions.

2.6.1 Notations

For i^{th} item (i = 1, 2, 3, …, n)

n - number of items
q_i - production quantity batch (decision variable) for the i^{th} item
R_i - annual demand rate for the i^{th} item
r_i - annual rate of production for the i^{th} item
P_i - unit purchase or production cost
C_i - unit holding cost per item
S_i - set up or ordering cost per order
$C_i(q_i)$ - unreliable holding cost for the i^{th} item
M_1 - limitation on the total number of orders
θ_i - percentage of imperfect items
$\tilde{P}_i$ - fuzzy production cost
$\tilde{L}_i$ - fuzzy lead time
$\tilde{TC}(q_i)$ - fuzzy total annual relevant cost

2.6.2 Assumptions

The following basic assumptions about the mathematical model are derived in our inventory problem.

- Demand rate is uniform over time.
- Shortages are not allowed.
- Lead time is allowed.
- Production cost and lead time are taken as a trapezoidal fuzzy number.
- The rate of production for each product is finite and constant.
- Transportation cost is not considered.
- Here we consider imperfect items, which are non-reusable and non-repairable items.
- The holding cost for the i^{th} item is a decreasing continuous function of the production quantity q_i and we are taken in the form is $C_i\left(q_i\right)=\alpha+\dfrac{\beta}{q_i}$,

  i = 1, 2,…, n, $\alpha>0$ and $\beta>0$ are real constants preferred to provide the best robust of the expected cost function, since $C_i\,(q_i)$ must be positive.
- Our aim is to minimize the annual pertinent total cost.

3. Model Formulation

3.1 Proposed Inventory Model in Fuzzy Sense

From the above notations and assumptions, we obtain the total annual relevant cost for the proposed inventory model in fuzzy sense.

Here, Production cost (P_i) and Lead time (L_i) are taken as a trapezoidal fuzzy number.

Let $\tilde{P}_i$ - fuzzy production cost

$\tilde{L}_i$ - fuzzy lead time

Then, the fuzzy total annual relevant cost of the EOQ model is given by,

$T\tilde{C}\,(q_i)$ = fuzzy production cost + setup cost + holding cost + fuzzy lead time + cost of the imperfect items

$$T\tilde{C}\,(q_i) = \sum_{i=1}^{n}\left\{R_i\,\tilde{P}_i+\frac{R_iS_i}{q_i}+\frac{q_i}{2}\left(1-\frac{R_i}{r_i}\right)C_i\left(q_i\right)+\tilde{L}_i\;+\theta_i\,q_i\right\} \tag{1}$$

Substituting $C_i\,(q_i)$ in equation (1), yields,

$$T\tilde{C}\left(q_i\right)=\sum_{i=1}^{n}\left\{R_i\,\tilde{P}_i+\tilde{L}_i\;\right\}+\sum_{i=1}^{n}\left\{\frac{R_i\,S_i}{q_i}+\frac{q_i}{2}\left(1-\frac{R_i}{r_i}\right)\left(\alpha+\beta\,q_i^{-1}\right)+\theta_i\,q_i\right\} \tag{2}$$

$$T\tilde{C}\,(q_i) = \sum_{i=1}^{n}\left\{R_i\,\tilde{P}_i\;+\tilde{L}_i+\frac{\beta}{2}\left(1-\frac{R_i}{r_i}\right)\right\}+\sum_{i=1}^{n}\left\{\frac{R_i\,S_i}{q_i}+\frac{\alpha\,q_i}{2}\left(1-\frac{R_i}{r_i}\right)+\theta_i\,q_i\right\} \tag{3}$$

There is a limitation on the total number of orders.

The constraint is,

$$\sum_{i=1}^{n}\frac{R_i}{q_i}\leq M_1 \tag{4}$$

where M_1 is the limit on total number of orders.

The term $\sum_{i=1}^{n}\left\{R_i\,\tilde{P}_i+\tilde{L}_i+\dfrac{\beta}{2}\left(1-\dfrac{R_i}{r_i}\right)\right\}$ is constant and hence we omit that term.

To solve this primal function which is a convex programming problem, let us write it in the subsequent simplified version of equation (3), then,

$$\min TC = \sum_{i=1}^{n}\left\{\frac{R_i\, S_i}{q_i} + \frac{\alpha R_i^{'}\, q_i}{2} + \theta_i\, q_i\right\} \tag{5}$$

Subject to the constraint

$$\sum_{i=1}^{n}\frac{R_i}{M_1\, q_i} \le 1 \quad , \quad \text{where } R_i^{'} = 1 - \frac{R_i}{r_i} \tag{6}$$

Apply the geometric programming technique to equations (5) and (6), we have the enlarged predual function might be written in the following form as,

$$G(\underline{W}) = \prod_{i=1}^{n}\left(\frac{R_i\, S_i}{W_{1i}}\right)^{W_{1i}}\left(\frac{\alpha\, R_i^{'}}{2W_{2i}}\right)^{W_{2i}}\left(\frac{\theta_i}{W_{3i}}\right)^{W_{3i}}\left(\frac{R_i}{M_1\, W_{4i}}\right)^{W_{4i}} q_i^{-W_{1i}+W_{2i}+W_{3i}-W_{4i}} \tag{7}$$

where the dual vector $(\underline{W}) = W_{ji}\,, 0 < W_{ji} < 1$, j = 1,2,3,4, i = 1, 2, 3,…, n is arbitrary and for convenience we choose the normality condition is,

$$W_{1i} + W_{2i} = 1 \tag{8}$$

We have to choose $(\underline{W})$ such that the exponent of q_i is zero, we make the R.H.S of equation (7) is independent of the decision variable.

So, we choose the orthogonality condition is,

$$-W_{1i} + W_{2i} + W_{3i} - W_{4i} = 0 \tag{9}$$

Then again, the problem is to select the minimum solution of the weights W_{ji}^{*} .

Solve equations (8) and (9), we get,

$$W_{1i} = \frac{1 + W_{3i} - W_{4i}}{2} \quad \text{and} \quad W_{2i} = \frac{1 - W_{3i} + W_{4i}}{2} \tag{10}$$

Substitute the value of W_{1i} and W_{2i} in equation (7), then the dual function is given by,

$$g\,(W_{3i}, W_{4i}) = \prod_{i=1}^{n}\left(\frac{2\,R_i S_i}{1 + W_{3i} - W_{4i}}\right)^{\left(\frac{1+W_{3i}-W_{4i}}{2}\right)}\left(\frac{\alpha\, R_i^{'}}{1 - W_{3i} + W_{4i}}\right)^{\left(\frac{1-W_{3i}+W_{4i}}{2}\right)}\left(\frac{\theta_i}{W_{3i}}\right)^{W_{3i}}\left(\frac{R_i}{M_1 W_{4i}}\right)^{W_{4i}} \tag{11}$$

Taking 'log' on both sides of equation (11), we get,

$$\begin{aligned}\log\left[g\,(W_{3i}, W_{4i})\right] = &\left(\frac{1 + W_{3i} - W_{4i}}{2}\right)\log(2R_i S_i) - \left(\frac{1 + W_{3i} - W_{4i}}{2}\right)\log(1 + W_{3i} - W_{4i}) + \\ &\left(\frac{1 - W_{3i} + W_{4i}}{2}\right)\log(\alpha R_i^{'}) - \left(\frac{1 - W_{3i} + W_{4i}}{2}\right)\log(1 - W_{3i} + W_{4i}) + \\ &W_{3i}\log(\theta_i) - W_{3i}\log(W_{3i}) + W_{4i}\log(R_i) - W_{4i}\log(M_1 W_{4i})\end{aligned} \tag{12}$$

Differentiate equation (12) partially w.r.to 'W_{3i}', we get,

$$\frac{\partial \log\left[g\,(W_{3i}, W_{4i})\right]}{\partial W_{3i}} = \frac{1}{2}\left[\log\left(\frac{2R_i S_i}{\alpha R_i^{'}}\right)\left(\frac{\theta_i}{e}\right)^2\left(\frac{1 - W_{3i} + W_{4i}}{1 + W_{3i} - W_{4i}}\right)\left(\frac{1}{W_{3i}^2}\right)\right] \tag{13}$$

Set the equation (13) to zero and simplifies, we get,

$$\left(\frac{2R_i S_i}{\alpha R_i^{'}}\right)\left(\frac{\theta_i}{e}\right)^2\left(\frac{1 - W_{3i} + W_{4i}}{1 + W_{3i} - W_{4i}}\right)\left(\frac{1}{W_{3i}^2}\right) = 1 \tag{14}$$

Differentiate equation (12) partially w.r.to 'W_{4i}', we get,

$$\frac{\partial \log[g(W_{3i},W_{4i})]}{\partial W_{4i}}=\frac{1}{2}\left[\log\left(\frac{\alpha R_i'}{2R_iS_i}\right)\left(\frac{R_i}{M_1e}\right)^2\left(\frac{1+W_{3i}-W_{4i}}{1-W_{3i}+W_{4i}}\right)\left(\frac{1}{W_{4i}^2}\right)\right] \qquad (15)$$

Set the equation (15) to zero and simplifies, we get,

$$\left(\frac{\alpha R_i'}{2R_iS_i}\right)\left(\frac{R_i}{M_1e}\right)^2\left(\frac{1+W_{3i}-W_{4i}}{1-W_{3i}+W_{4i}}\right)\left(\frac{1}{W_{4i}^2}\right)=1 \qquad (16)$$

Multiplying relation (14) by the relation (16), we have

$$W_{3i}\,W_{4i} = \frac{\theta_i\,R_i}{M_1e^2} \qquad (17)$$

From equation (17), we get the value of W_{3i} and W_{4i},

$$W_{3i} = \frac{\theta_i\,R_i}{M_1e^2\,W_{4i}} \quad \text{and}$$

$$W_{4i} = \frac{\theta_i\,R_i}{M_1e^2\,W_{3i}} \qquad (18)$$

Substitute the value of W_{4i} in the equation (14), we get,

$$f(W_{3i})=W_{3i}^4+W_{3i}^3-\left[\left(\frac{\theta_iR_i}{M_1e^2}\right)-\left(\frac{2R_iS_i}{\alpha R_i'}\right)\left(\frac{\theta_i}{e}\right)^2\right]W_{3i}^2-\left(\frac{2R_iS_i}{\alpha R_i'}\right)\left(\frac{\theta_i}{e}\right)^2W_{3i}$$
$$-\left(\frac{2R_iS_i}{\alpha R_i'}\right)\left(\frac{\theta_i}{e}\right)^2\left(\frac{\theta_iR_i}{M_1e^2}\right)=0$$

$$\Rightarrow f(W_{3i})=W_{3i}^4+W_{3i}^3-(A_i-B_{3i})W_{3i}^2-B_{3i}W_{3i}-A_i\,B_{3i}=0 \qquad (19)$$

where $A_i = \left(\frac{\theta_iR_i}{M_1e^2}\right)$ and $B_{3i} = \left(\frac{2R_iS_i}{\alpha R_i'}\right)\left(\frac{\theta_i}{e}\right)^2$

Substitute the value of W_{3i} in the equation (16), we get,

$$f(W_{4i})=W_{4i}^4+W_{4i}^3-\left[\left(\frac{\theta_iR_i}{M_1e^2}\right)-\left(\frac{\alpha R_i'}{2R_iS_i}\right)\left(\frac{R_i}{M_1e}\right)^2\right]W_{4i}^2-\left(\frac{\alpha R_i'}{2R_iS_i}\right)\left(\frac{R_i}{M_1e}\right)^2W_{4i}$$
$$-\left(\frac{\alpha R_i'}{2R_iS_i}\right)\left(\frac{R_i}{M_1e}\right)^2\left(\frac{\theta_iR_i}{M_1e^2}\right)=0$$

$$\Rightarrow f(W_{4i})=W_{4i}^4+W_{4i}^3-(A_i-B_{4i})W_{4i}^2-B_{4i}W_{4i}-A_i\,B_{4i}=0 \qquad (20)$$

where $A_i = \left(\frac{\theta_iR_i}{M_1e^2}\right)$ and $B_{4i} = \left(\frac{\alpha R_i'}{2R_iS_i}\right)\left(\frac{R_i}{M_1e}\right)^2$

Combining equation (19) and (20), we get,

$$\Rightarrow f\left(W_{ji}\right)=W_{ji}^{4}+W_{ji}^{3}-\left(A_{i}-B_{ji}\right)W_{ji}^{2}-B_{ji}W_{ji}-A_{i}B_{ji}=0 \quad j=3,4 \qquad (21)$$ where

$$A_{i}=\left(\frac{\theta_{i}R_{i}}{M_{1}e^{2}}\right),\ B_{3i}=\left(\frac{2R_{i}S_{i}}{\alpha R_{i}^{'}}\right)\left(\frac{\theta_{i}}{e}\right)^{2} \text{ and } B_{4i}=\left(\frac{\alpha R_{i}^{'}}{2R_{i}S_{i}}\right)\left(\frac{R_{i}}{M_{1}e}\right)^{2}$$

Now,

$$f\left(0\right)=-A_{i}B_{ji}<0,\ j=3,4$$

$$f\left(1\right)=2-A_{i}(1+B_{ji})>0,\ j=3,4$$

Therefore, a root lies between 0 and 1 and j = 3, 4.

For calculation, we use the trial and error method.

We shall first verify that any root W_{ji}^{*}, j = 3,4 calculated from equation (21) maximizes $g(W_{ji}^{*})$, j = 3, 4.

This is simply possible by finding the second derivative of the equations (13) and (15).

$$\frac{\partial^{2}\log\left(W_{3i},W_{4i}\right)}{\partial W_{3i}^{2}}<0,\ \frac{\partial^{2}\log\left(W_{3i},W_{4i}\right)}{\partial W_{4i}^{2}}<0 \text{ and } \frac{\partial^{2}\log\left(W_{3i},W_{4i}\right)}{\partial W_{3i}\partial W_{4i}}>0.$$

Hence, $\Delta=\left(\frac{\partial^{2}\log\left(W_{3i},W_{4i}\right)}{\partial W_{3i}\partial W_{4i}}\right)^{2}-\left(\frac{\partial^{2}\log\left(W_{3i},W_{4i}\right)}{\partial W_{3i}^{2}}\right)\left(\frac{\partial^{2}\log\left(W_{3i},W_{4i}\right)}{\partial W_{4i}^{2}}\right)<0.$

Thus, the roots W_{3i}^{*} and W_{4i}^{*} are calculated from equation (21) maximize the dual function g (W_{3i}, W_{4i}).

Hence the minimum solution is W_{ji}^{*}, j = 1,2,3,4, where W_{3i}^{*}, W_{4i}^{*} are the solutions of equation (21) and W_{1i}^{*}, W_{2i}^{*} are calculated by substituting the values of W_{3i}^{*} and W_{4i}^{*} in the equation (10).

To calculate the optimal order quantity q_{i}^{*}, we apply Duffin and Peterson's theorem of geometric programming as:

$$\frac{R_{i}S_{i}}{q_{i}^{*}}=W_{1i}^{*}\,g\left(W_{1i}^{*},W_{2i}^{*},W_{3i}^{*}\right)$$

$$\frac{\alpha R_{i}^{'}q_{i}^{*}}{2}=W_{2i}^{*}\,g\left(W_{1i}^{*},W_{2i}^{*},W_{3i}^{*}\right) \text{ and}$$

$$\theta_{i}\,q_{i}^{*}=W_{3i}^{*}\,g\left(W_{1i}^{*},W_{2i}^{*},W_{3i}^{*}\right)$$

Solving the above relations, then we obtain the optimal order quantity is,

$$\Rightarrow q_{i}^{*}=\sqrt{\frac{2R_{i}S_{i}W_{2i}^{*}}{\alpha R_{i}^{'}W_{1i}^{*}}},\ i=1,2,3,\ldots,n \qquad (22)$$

Substitute equation (22) in equation (3), we get,

$$\min T\tilde{C} = \sum_{i=1}^{n}\left[R_i\,\tilde{P}_i + \frac{\beta R_i^{'}}{2} + \tilde{L}_i + \sqrt{\frac{\alpha R_i R_i^{'} S_i}{2W_{1i}^{*} W_{2i}^{*}}} + \theta_i \sqrt{\frac{2R_i S_i W_{2i}^{*}}{\alpha R_i^{'} W_{1i}^{*}}}\right] \tag{23}$$

4. Numerical Example

4.1 Numerical Example in Fuzzy Sense

The annual demand rate of an item is 300 units, the annual rate of production of an item is 500 units, unit purchase or production cost per item is Rs. (15, 20, 25, 30), set up or ordering cost per order is Rs. 400. If there is 5 % of imperfect items and the lead time is (0.1, 0.2, 0.3, and 0.4). Assume that the total number of orders for each year is 1500, α = Rs. 1 and β = 0.5. The order quantity and minimum total annual cost are to be determined.

Sol:

n = 1
R = 300 units
P = Rs. (15, 20, 25, 30)
r = 500 units
S = Rs. 400 / unit
L = (0.1, 0.2, 0.3, 0.4)
θ = 5 %
M_1 = 1500 ft^2
α = Rs. 1 / unit
β = 0.5

Order Quantity

$$q^{*} = \left[\frac{2\,R\,S\,W_2^{*}}{\alpha\,R^{'}\,W_1^{*}}\right]^{1/2}$$

$$q^{*} = 755$$

Fuzzy minimum annual total relevant cost

$$\min T\tilde{C} = RP + \frac{\beta R^{'}}{2} + \tilde{L} + \sqrt{\frac{\alpha R R^{'} S}{2W_1^{*} W_2^{*}}}$$

$$\min T\tilde{C} = \text{Rs. } (4847.79,\ 6347.89,\ 7847.99,\ 9348.09)$$

Graded Mean Representation Method

$$P\left(T\tilde{C}\right) = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6}$$

$$P\left(T\tilde{C}\right) = \text{Rs. } 7097.94$$

5. Sensitivity Analysis

A sensitivity analysis of this fuzzy model has been derived for varying β value. Here we consider an inventory model for five items with the following input data.

Table: 1

N	R (units)	r (units)	P (Rs.)	S (Rs.)	$\tilde{L}$	α (Rs.)	θ
1	300	500	(15, 20, 25, 30)	400	(0.1,0.2,0.3,0.4)	1	5 %
2	250	400	(15, 20, 25, 30)	300	(0.1,0.2,0.3,0.4)	1	5 %
3	200	300	(15, 20, 25, 30)	200	(0.1,0.2,0.3,0.4)	1	5 %
4	150	200	(15, 20, 25, 30)	100	(0.1,0.2,0.3,0.4)	1	5 %
5	100	100	(15, 20, 25, 30)	50	(0.1,0.2,0.3,0.4)	1	5 %

Assume that the total number of order for each year is 1500 ft^2 .

Table: 2

S.No	β	q_1^*	q_2^*	q_3^*	q_4^*	q_5^*	$C_1(q_1^*)$	$C_2(q_2^*)$	$C_3(q_3^*)$	$C_4(q_4^*)$	$C_5(q_5^*)$	$\min(T\tilde{C})$
1	0.1	755	616	478	338	98	1.0001	1.0002	1.0002	1.0003	1.0010	23,512.53
2	0.5	755	616	478	338	98	1.0007	1.0008	1.0010	1.0015	1.0051	23,513.00
3	1	755	616	478	338	98	1.0013	1.0016	1.0021	1.0030	1.0102	23,513.60
4	2	755	616	478	338	98	1.0026	1.0032	1.0042	1.0059	1.0204	23,514.77
5	5	755	616	478	338	98	1.0067	1.0081	1.0105	1.0148	1.0510	23,518.31
6	10	755	616	478	338	98	1.0132	1.0162	1.0209	1.0296	1.1020	23,524.21
7	50	755	616	478	338	98	1.0662	1.0812	1.1046	1.1479	1.5102	23,571.37
8	100	755	616	478	338	98	1.1325	1.1623	1.2092	1.2959	2.0204	23,630.33
9	200	755	616	478	338	98	1.2649	1.3247	1.4184	1.5917	3.0408	23,748.24
10	500	755	616	478	338	98	1.6623	1.8117	2.0460	2.4793	6.1020	24,102.00

6. Conclusion

This paper investigates to find a minimum total annual relevant cost for imperfect quality items with lead time and unreliable holding cost using geometric programming method in fuzzy sense. Production cost and lead time are taken as a trapezoidal fuzzy numbers. Finally, the proposed model has been verified by the numerical example along with the sensitivity analysis. In the sensitivity analysis we obtain the optimal solution for varying β . From the above table, it implies that the dealers will reduce the value of β , and then only we get the minimum total annual relevant cost. Further, future research work can be done by extending the developed model for more realistic situations with time varying holding costs and varying production cost etc.

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